ECE4390 Lab 4

The purpose of this lab is to familiarize the student with the Finite Element Method (FEM). This computational technique will be applied to solve the 2D Laplace ($\nabla^2\Phi(x, y) = 0$) and Poisson ($\nabla^2\Phi(x, y) = -\rho/\epsilon$) equations for different domains. Discretizations of these domains are provided on the course website in the form of GMSH meshes. For those who are interested, GMSH is is available for free at http://geuz.org/gmsh/ and is distributed under the GNU General Public License. A good introductory GMSH tutorial is available at http://ffep.sourceforge.net/Download/gui_tutorial.pdf.

A report is to be handed-in. All the requested plots are to be submitted and all questions answered. Be sure to include any necessary explanations describing the plots. Also, submit a print-out of your code. But if you wish to save paper, the files can be emailed to umfauch2@cc.umanitoba.ca. Be sure to include your name and student number. There will also be a 15 to 20 minute quiz to be written near the end of the lab period.

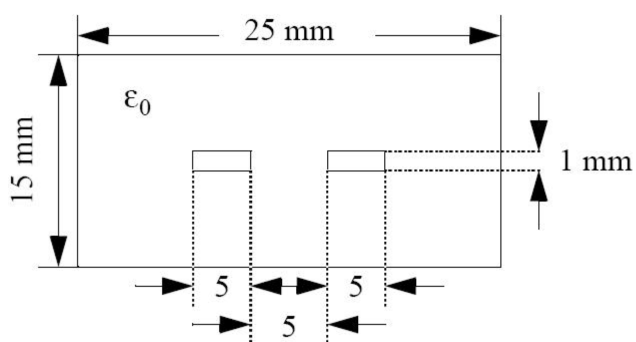
Part 1: Familiarization with the FEM code

1a. Download the laboratory files from the course website into a single folder. Start by running the "electrostatic_fem_probe.m" file in MATLAB. Observe the result. Go through the source code and locate the lines that set the Dirichlet boundary condition values. Spend some time to understand how the global system of equations is created to solve Laplace's equation from the given mesh. Note how mesh element groups are given an identification tag (physics number) to differentiate them from other element groups. For instance, the nodes on the probe lines are given a physics number of 100 to differentiate them from the nodes on the domain boundary having physics numbers of 101 and 102.

1b. Run the program for the same domain but with the upper boundary set to a Dirichlet condition of $\Phi = 50$ (hint: the upper boundary is tagged with a physics number of 102). Make any necessary modifications to the "electrostatic_fem_probe.m" program to handle this new boundary and find the solution to this new domain. Plot the resultant solution with the above modifications.

Part 2: Solution of Laplace's equation using FEM

GMSH meshes are provided for the domain shown below (files:"txline.msh" and "txline_refined.msh"). In both meshes, the outer boundary physics number is set to 100, the left inner-box boundary to 200 and the other to 201.



2a. As was done in the previous lab, set boundary conditions on the inner boxes to represent two conductors. Set the left conductor to 1V and the other to ground. Solve Laplace's equation over the domain described above with the outer boundary set to $\Phi = 0$ (use mesh named "txline.msh"). Plot the result.

2b. Use the FEM solution to determine the p.u.l. capacitance of the transmission line. This is done by applying the integral form of Gauss's law for a contour of your choice about the left conductor. Use the contour integration method from the previous lab as a guideline for this case. Give the details about how you decided to implement this integral. Your result should be comparable to the p.u.l. capacitance found in the previous lab. Hint: Recall from class notes that $\Phi(x, y)$ for (x, y) in the problem domain is approximated by $\phi^{(e)}(x, y) = \underline{\phi}^T \underline{\alpha}(x, y)$ where e is the element that contains the point (x, y), $\underline{\phi}$ is a column vector of solution values at the vertices of element e and $\underline{\alpha}$ is the column vector of shape functions for element e. To find the element that contains a particular point, the provided MATLAB function "getEleIndices.m" can be used. It requires two input parameters. The first is the struct that is returned by the call to "GmshReadM.m". The second parameter is a $p \times 2$ matrix where p is the number of points to locate and each point is described by its x-coordinate in the first column and its y-coordinate in the second column. The function then returns a $p \times 1$ vector of element indices in the order of corresponding points.

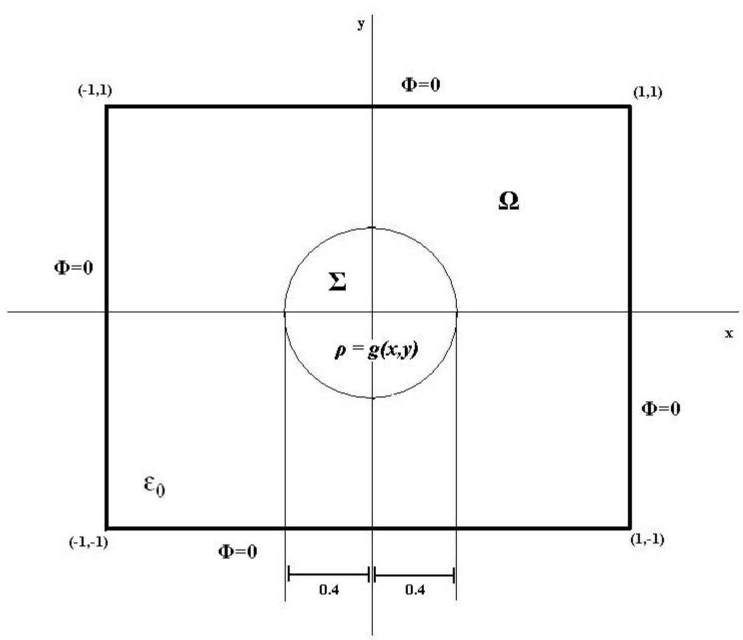
2c. Solve Laplace's equation once more over the same domain but this time use a more refined mesh (use mesh named "txline_refined.msh"). The refined mesh should produce a more accurate solution. Describe in your own words why this should be the case. Determine the p.u.l. capacitance of the transmission line for this new, more accurate result. Your result should be a very close match to the p.u.l. capacitance found in the previous lab.

Part 3: Solution of Poisson's equation using FEM

The geometry shown below represents a circle with charge distribution $g(x, y)$ enclosed by a grounded boundary. No free charge is imposed outside of the circle. The imposed charge distribution is given by

$$\rho = g(x, y) = \begin{cases} 100e^{-(100x^2+100y^2)} & \text{if } (x, y) \in \Sigma \\ 0 & \text{otherwise} \end{cases}$$

A GMSH mesh is provided for the domain shown below (file:"charged_circ.msh"). The outer boundary physics number is set to 100, the Ω region (outside the circle) to 200 and the Σ region (inside the circle) to 201.



3a. Modify the given FEM program to solve Poisson's equation over the above domain such that the forcing function $\rho = g(x, y)$ is assumed to be constant over a particular element. For each element, $g(x, y)$ should be evaluated at the element centroid. Plot the solution. Determine the total charge on the plate by applying the integral form of Gauss's law. Hint: The 2D element centroid coordinates are available in the form of two vectors in the "MeshData" struct that is returned from the call to "GmshReadM.m". The vectors of x and y coordinates are named "xCentroids" and "yCentroids", respectively. In these vectors, the coordinates are ordered by element index. The 2D element physics numbers are also available in the "MeshData" struct. They are ordered by element index in the "PhysNumber" vector. It is also useful to note that the integral of a shape function over its element is equal to a third of the element's area.

3b. Modify the given FEM program to solve Poisson's equation over the same domain such that the forcing function $\rho = g(x, y)$ is expanded with shape functions over each element (as opposed to remaining constant). Plot the solution. Determine the total charge on the plate by applying the integral form of Gauss's law.

3c. Compare the two computed solutions. Which is more accurate and why?